20-2 Geometric Series

1. $S_{10} = 40\left(\frac{1-\frac{1}{2}^{10}}{1-\frac{1}{2}}\right) = 79.921875$

2. $S_{15} = 4\left(\frac{1-3^{15}}{1-3}\right) = 28,697,812$

3. $S_{40} = 15\left(\frac{1-\frac{4}{5}^{40}}{1-\frac{4}{5}}\right) = 74.99$

4. $S_{100} = 27\left(\frac{1-(\frac{1}{3})^{100}}{1-\frac{1}{3}}\right) = 40.5$

5. $S_8 = 0.2\left(\frac{1-0.1^8}{1-0.1}\right) = 0.22\overline{2}$

6. $S_6 = 100\left(\frac{1-2^6}{1-2}\right) = 6300$

7. $S_{12} = 400\left(\frac{1-1.1^{12}}{1-1.1}\right) = \$8553.71$

400, 440, 484, ...

8. $3 + \frac{3}{2} + \frac{3}{4} + \ldots$ $r = \frac{1}{2}$ Infinite sum: $\frac{a_1}{1-r}$

converges → 6

9. $4 + 2 + 1 \ldots$ $r = \frac{1}{2}$

converges → 8

10. $17 + 15.3 + 13.77 + \ldots$ $r = 0.9$

converges → 170

11. $6 + 11.4 + 21.66 + \ldots$ $r = 1.9$

diverges → No ∞ sum

12. $-20 + (-8) + (-3.2) + \ldots$ $r = 0.4$

converges → $-33.\overline{3}$

13. $50 + 70 + 98 + \ldots$ $r = 1.4$

diverges → no ∞ sum

14. $8 + 4 + 2 + 1 + \ldots$ $r = \frac{1}{2}$

$$\frac{8}{1 - \frac{1}{2}} = 16$$

15. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots \quad r = \frac{1}{3}$

$$\frac{1}{1 - \frac{1}{3}} = 1.5$$

16. $120 + 96 + 76.8 + 61.44 + \ldots \quad r = 0.8$

$$\frac{120}{1 - 0.8} = 600$$

17. $100 + 750 + 562.5 + 421.875 + \ldots \quad r = 0.75$

$$\frac{1000}{1 - 0.75} = 4000$$

18. $a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$5500, \ 6600, \ 7920, \ 9504, \ 11404.80$

$$S_5 = 5500\left(\frac{1 - 1.2^5}{1 - 1.2}\right) = \$40,928.80$$

$r = 1.2$ ← increase by 20%.

19. $a_1 = 50$ +

$a_2 = 49.5$ +

$a_3 = 49.005$

$\vdots$ ↓ ∞ sum

$a_n = 0$

$$S = \frac{50}{1 - 0.99} = 5000 \text{ cm}$$

20. $32\text{ mm}, \quad 26.24, \quad 21.5168, \ldots$

$$S = \frac{32}{1-0.82} = 177.78\text{ mm total}$$

21. $495{,}000, \quad 445{,}500, \quad 400{,}950, \ldots \quad r = 0.9$

a) $$S_6 = 495{,}000\left(\frac{1-0.9^6}{1-0.9}\right) = \$2{,}319{,}367.05$$

b) $$S = \frac{495{,}000}{1-0.9} = \$4{,}950{,}000$$

↖ as long as people will buy $\to \infty$

22. $2+5+8+11+\ldots \quad n=9 \quad$ arithmetic

$d=3$

$$S_9 = \frac{9}{2}(2+26) = 126$$

$$a_9 = 2 + (9-1)(3)$$

$$a_9 = 26$$

23. $\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \ldots \quad n=8 \quad$ geometric

$r = \frac{1}{2}$

$$S_8 = \frac{1}{8}\left(\frac{1-\frac{1}{2}^8}{1-\frac{1}{2}}\right) = .249023 \text{ or } \frac{255}{1024}$$

24. $-3 + 6 - 12 + 24 - \ldots \quad n = 10$ geometric

$r = -2$

$$S_{10} = -3\left(\frac{1-(-2)^{10}}{1-{-2}}\right) = 1023$$

25. $-2 + 2 + 6 + 10 + \ldots \quad n = 12$ arithmetic

$d = 4$

$$S_{12} = \frac{12}{2}(-2 + 42) = 240$$

$$a_{12} = -2 + (12-1)(4) = 42$$

26. $4 + 8 + 16 + 32 + \ldots \quad n = 15$ geometric

$r = 2$

$$S_{15} = 4\left(\frac{1-2^{15}}{1-2}\right) = 131{,}068$$

27. $5 + 10 + 15 + 20 + \ldots \quad n = 20$ arithmetic

$d = 5$

$$S_{20} = \frac{20}{2}(5 + 100) = 1050$$

$$a_{20} = 5 + (20-1)(5)$$
$$= 100$$

28. $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1} \quad r = \frac{2}{3}$ converges

$$S = \frac{5}{1 - \frac{2}{3}} = 15$$

29. $\sum_{n=1}^{\infty} (-2.1)^{n-1}$ $a_1 = 1$ $r = -2.1$ Diverges → no infinite sum

30. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$ $a_1 = 1$ $r = -\frac{1}{2}$ Converges

$$S = \frac{1}{1 - -\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

31. $\sum_{n=1}^{\infty} 2\left(\frac{5}{3}\right)^{n-1}$ $a_1 = 2$ $r = \frac{5}{3}$ Diverges → no infinite sum

33. $a_1 = 5$ $S = \frac{25}{3}$ find r.

$$\frac{25}{3} = \frac{5}{1-r}$$

$$25 - 25r = 15$$

$$-25r = -10$$

$$r = \frac{10}{25}$$

$$r = \frac{2}{5}$$

34. $S = 108$ $r = \frac{1}{3}$ $a_1 = ?$

$$108 = \frac{a_1}{1 - \frac{1}{3}} \rightarrow \frac{2}{3} \cdot 108 = \frac{a_1}{\frac{2}{3}} \cdot \frac{2}{3}$$

$$72 = a_1$$

36. $S = 840 \quad r = 0.5 \quad a_1 = ?$

$$840 = \frac{a_1}{1 - 0.5}$$

$$840 = \frac{a_1}{0.5}$$

$$420 = a_1$$