Unit 3 Test Review Solutions

1. $f(x) = -4x^6 + 7x^3 + 5x^2 - 2x + 4$

   Degree: 6

   Leading Coefficient: -4

2. $(4x^2 - 2 + 5x) + 4(3x - 8x^2 - 5)$

   $4x^2 - 2 + 5x + 12x - 32x^2 - 20$

   $-28x^2 + 17x - 22$

3. $2(7x^2 - 8x - 8) - 4(8 + 2x^2 + 5x)$

   $14x^2 - 16x - 16 - 32 - 8x^2 - 20x$

   $6x^2 - 36x - 48$

4. $(a-2)(3a^2 + 6a + 1)$

   $3a^3 + 6a^2 + a$

   $-6a^2 - 12a - 2$

   $3a^3 - 11a - 2$

5. $(4n-5)^2(7n^2+4)$

   $(4n-5)(4n-5)(7n^2+4)$

   $(16n^2 - 20n - 20n + 25)(7n^2+4)$

   $(16n^2 - 40n + 25)(7n^2+4)$

   $112n^4 + 64n^2 - 280n^3 - 160n + 175n^2 + 200$

   $112n^4 - 280n^3 + 239n^2 - 160n + 200$

6. $(x^4 - 2x^3 + x^2 + x - 1) \div (x - 1)$

$$\begin{array}{r|ccccc} 1 & 1 & -2 & 1 & 1 & -1 \\ & & 1 & -1 & 0 & 1 \\ \hline & 1 & -1 & 0 & 1 & \text{OR} \end{array}$$

$= x^3 - x^2 + 1$

7. $(x^3 - 9x^2 - 1) \div (x - 9)$

$$\begin{array}{r|ccccc} 9 & 1 & -9 & 0 & -1 \\ & & 9 & 0 & 0 \\ \hline & 1 & 0 & 0 & -1^R \end{array}$$

$= 1x^2 + \frac{-1}{x-9}$

8. $\dfrac{4x^3 + 21x^2 - x - 24}{x + 5}$

$$\begin{array}{r|cccc} -5 & 4 & 21 & -1 & -24 \\ & & -20 & -5 & 30 \\ \hline & 4 & 1 & -6 & 6^R \end{array}$$

$= 4x^2 + x - 6 + \frac{6}{x+5}$

9. $245k^4 + 35k^3 + 49k^2 + 7k$

$7k((35k^3 + 5k^2)(+7k + 1))$

$\downarrow \; 5k^2(7k+1)\;1\;(7k+1)$

$= 7k(7k+1)(5k^2+1)$

10. $x^8 - 17x^4 + 16$

$(x^4 - 16)(x^4 - 1)$

$(x^2+4)(x^2-4)(x^2+1)(x^2-1)$

$= (x^2+4)(x+2)(x-2)(x^2+1)(x+1)(x-1)$

11. $x^7 - x = 0$

$x(x^6 - 1)$ ← DOTS

$x(x^3+1)(x^3-1)$ ← Sum & Diff of cubes

$= x(x+1)(x^2-x+1)(x-1)(x^2+x+1)$

12. $f(x) = 27x^6 + 28x^3 + 1$

$= (27x^3 + 1)(x^3 + 1)$ ← Both Sum of cubes

$= (3x+1)(9x^2-3x+1)(x+1)(x^2-x+1)$

$3x+1=0$ → $\boxed{x = -\frac{1}{3}}$

$x+1=0$ → $\boxed{x=-1}$

$9x^2-3x+1=0$

$x = \frac{3 \pm \sqrt{9-4(9)(1)}}{2(9)}$

$x = \frac{3 \pm \sqrt{-27}}{18}$

$x = \frac{3 \pm 3i\sqrt{3}}{18}$

$\boxed{x = \frac{1 \pm i\sqrt{3}}{6}}$

$x^2-x+1=0$

$x = \frac{1 \pm \sqrt{1-4(1)(1)}}{2(1)}$

$x = \frac{1 \pm \sqrt{-3}}{2}$

$\boxed{x = \frac{1 \pm i\sqrt{3}}{2}}$

13. $f(x) = x^4 + 8x$

$= x(x^3 + 8)$

$= x(x+2)(x^2 - 2x + 4)$

$\boxed{x = 0}$ $\quad x + 2 = 0$ $\quad x^2 - 2x + 4 = 0$

$\boxed{x = -2}$ $\quad x = \frac{2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-12}}{2}$

$x = \frac{2 \pm 2i\sqrt{3}}{2} \rightarrow \boxed{x = 1 \pm i\sqrt{3}}$

14. $x = 0 \quad x = 3 \quad x = 2i \quad x = -2i$

$x(x-3)(x-2i)(x+2i)$

$(x^2 - 3x)(x^2 + 4)$

$x^4 + 4x^2 - 3x^3 - 12x$

$= x^4 - 3x^3 + 4x^2 - 12x$

15. $x = 1 \quad x = -5 \quad x = -5$

$(x-1)(x+5)(x+5)$

$(x-1)(x^2 + 10x + 25)$

$x^3 + 10x^2 + 25x$

$\quad - x^2 - 10x - 25$

$= x^3 + 9x^2 + 15x - 25$

16. a) $f(x) = 2x^4 - 3x^3 - x - 6$

p: 1, 2, 3, 6

q: 1, 2

Possible Rational Roots: $\pm 1 \pm \frac{1}{2} \pm 2 \pm 3 \pm \frac{3}{2} \pm 6$

$x = 2, \quad x = -1$

$x = \frac{1 \pm i\sqrt{23}}{4}$

2 |	2	-3	0	-1	-6
		4	2	4	6
-1 |	2	1	2	3	OR
		-2	1	-3	
	2	-1	3	OR	

$2x^2 - x + 3 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(2)(3)}}{2(2)}$

$x = \frac{1 \pm \sqrt{-23}}{4} = \frac{1 \pm i\sqrt{23}}{4}$

b) $f(x) = x^3 - 10x - 12$

p: 1, 2, 3, 4, 6, 12

q: 1

Possible Rational Roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$x = -2$

-2 |	1	0	-10	-12
		-2	4	12
	1	-2	-6	0

$x^2 - 2x - 6 = 0$

$x = \frac{2 \pm \sqrt{4 - 4(1)(-6)}}{2}$ $x = \frac{2 \pm \sqrt{28}}{2}$ →

16b continued... $x = \frac{2 \pm \sqrt{28}}{2}$

$x = \frac{2 \pm 2\sqrt{7}}{2}$

$x = 1 \pm \sqrt{7}$ $x = -2$

17. a) $f(x) = 3x^3 - 4x^2 - 26x + 33$ p: 1, 3, 11, 33

q: 1, 3

Possible Rational Roots: $\pm 1 \pm \frac{1}{3} \pm 3 \pm 11 \pm \frac{11}{3} \pm 33$

y-intercept: (0, 33)

end behavior: $x \to -\infty$, $y \to -\infty$ * See graph on last page

$x \to \infty$, $y \to \infty$

Roots: $x = 3$

$x \approx 1.25$

$x \approx -2.92$

3 | 3 -4 -26 33

9 15 -33

3 5 -11 OR

$3x^2 + 5x - 11$

$x = \frac{-5 \pm \sqrt{9 - 4(3)(-11)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{141}}{6}$

$x \approx 1.25$ $x \approx -2.92$

17. b) $f(x) = x^3 - 8x^2 - 16x + 8$

Possible Rational Roots: $\pm 1, \pm 2, \pm 4, \pm 8$

y-intercept: $(0, 8)$

end behavior: $x \to -\infty, y \to -\infty$

$x \to \infty, y \to \infty$

Roots: $x = -2$

$x \approx 0.42$

$x \approx 9.58$

$-2 \mid 1 \quad -8 \quad -16 \quad 8$

$\quad\quad -2 \quad 20 \quad -8$

$1 \quad -10 \quad 4 \quad$ OR

$x^2 - 10x + 4 = 0$

$$x = \frac{10 \pm \sqrt{100 - 4(1)(4)}}{2}$$

$$x = \frac{10 \pm \sqrt{84}}{2}$$

$$x = \frac{10 \pm 2\sqrt{21}}{2} = 5 \pm \sqrt{21}$$

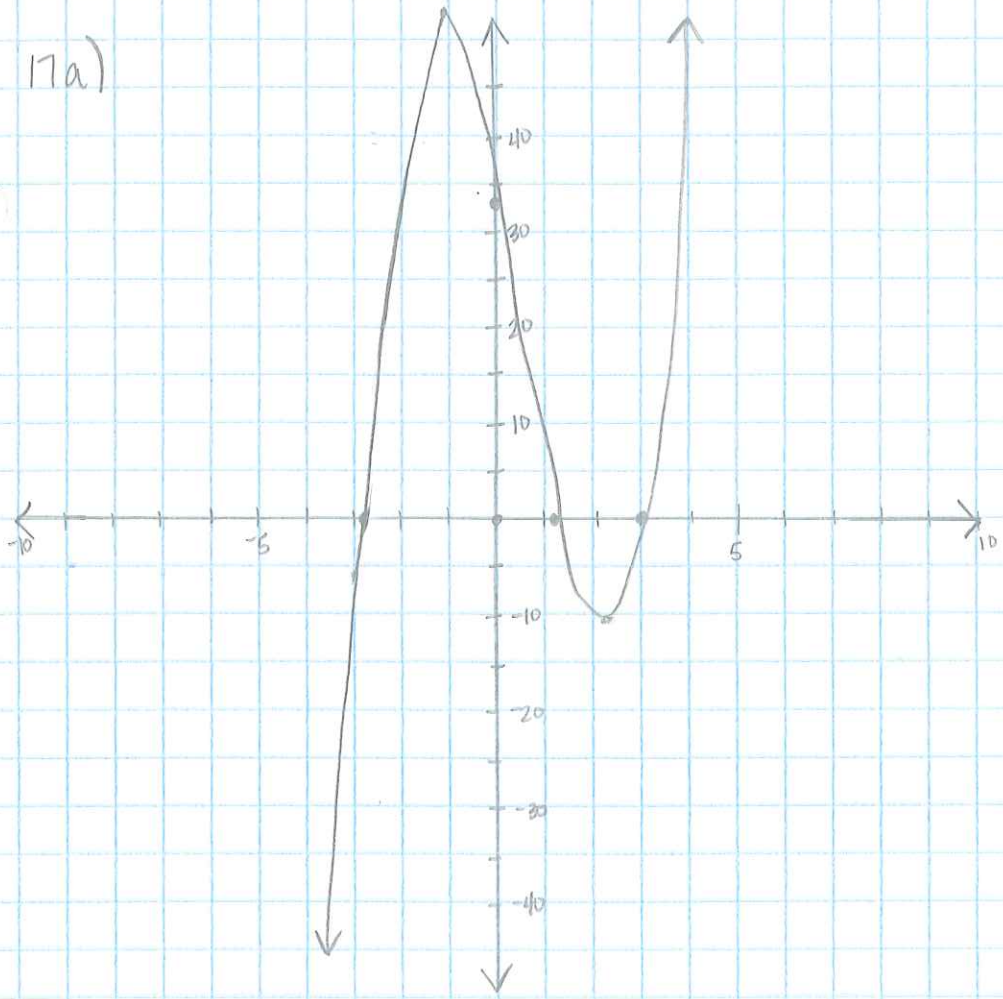
$x \approx 9.58 \quad x \approx 0.42$

$\sqrt{84}$

$4 \quad 21$

* See graph on last page

17a)



17b)